Many-body effects between unbosonized excitons

M. Combescot and O. Betbeder-Matibet

GPS, Université Pierre et Marie Curie, CNRS,

Campus Boucicaut, 140 rue de Lourmel, 75015 Paris, France

Abstract

We here give a brief survey of our new many-body theory for composite excitons, as well as some of the results we have already obtained using it. In view of them, we conclude that, in order to fully trust the results one finds, interacting excitons should not be bosonized: Indeed, all effective bosonic Hamiltonians (even the hermitian ones !) can miss terms as large as the ones they generate; they can even miss the dominant term, as in problems dealing with optical nonlinearities.

We have recently developed a new many-body theory for interacting composite bosons [1-8], which is actually an important break-through: Indeed, up to now, the existing many-body theories were appropriate to interactions between fermions or bosons [9-11]. Although essentially all bosons known in physics are composite bosons, there were no many-body procedures adapted to them, most probably because it is far more subtil to handle composite bosons properly than to handle true bosons or true fermions.

In order to "do something" in problems in which interactions between composite bosons obviously play a role, it is a common practice to replace composite bosons by true bosons, provided that their interactions are "dressed by exchange" – which is considered as a way to take care of their composite nature. The production of such effective bosonic Hamiltonians is the aim of the wide literature on bosonization [12,13].

This pragmatic idea is after all perfectly acceptable. The real (major) trouble in semiconductor physics, is that the effective bosonic Hamiltonian which has been produced [14,15] – and widely used – to treat interactions between excitons, should have been rejected long ago because it is not even hermitian: This failure, which comes from unconsistent manipulations in producing this effective Hamiltonian, has apparently been missed by everyone for 35 years ! As we were the first to point it out, it is easy to understand why our new theory for interacting composite excitons has not been welcomed by the experts in the field.

With a first line – the Hamiltonian – physically unacceptable, there is no reason for the last line – the deduced results – to be correct, except if some cancellations in the missed terms miraculously take place. Unfortunately, they don't ! Even if hermiticity is forced – which is always possible – essentially the same amount of terms is missing. In all the physical quantities we have studied up to now using our new theory for composite excitons, the missed terms are as large as the ones which appear. In some cases, in particular the ones in which photons are involved, *i. e.*, in optical nonlinearities, the missed terms can even be the dominant ones, as obvious from a dimensional argument explained below. Excitons are deeply made of two fermions; there is no way to get rid of their composite nature, as we do when we bosonize them.

Survey of our new many-body theory

Our new many-body theory for composite bosons relies on four nicely simple commu-

tators [1,3]:

$$\left[H, B_i^\dagger\right] = E_i B_i^\dagger + V_i^\dagger \ , \tag{1}$$

$$\left[V_i^\dagger, B_j^\dagger\right] = \sum_{mn} \xi_{\rm dir}\begin{pmatrix} n & j \\ m & i \end{pmatrix} B_m^\dagger B_n^\dagger \ , \tag{2}$$

$$\left[B_m, B_i^\dagger\right] = \delta_{m,i} - D_{mi} \ , \tag{3}$$

$$\left[D_{mi}, B_j^\dagger\right] = \sum_n \left[\lambda_h\begin{pmatrix} n & j \\ m & i \end{pmatrix} + \lambda_e\begin{pmatrix} n & j \\ m & i \end{pmatrix}\right] B_n^\dagger \ . \tag{4}$$

In the case of excitons, H is the semiconductor Hamiltonian, $B_i^\dagger$ the creation operator of *one* exciton in the i state, *i. e.*, $(H - E_i)B_i^\dagger|v\rangle = 0$. In terms of free electrons and holes, it reads

$$B_i^\dagger = \sum_{\mathbf{k}_e,\mathbf{k}_h} \langle \mathbf{k}_e, \mathbf{k}_h|\phi_i\rangle \, a_{\mathbf{k}_e}^\dagger b_{\mathbf{k}_h}^\dagger \ , \tag{5}$$

where $\langle \mathbf{k}_e, \mathbf{k}_h|\phi_i\rangle$ is the i exciton wave function in momentum space.

The above commutators generate two "scatterings":

(i) The first one $\xi_{\rm dir}\begin{pmatrix} n & j \\ m & i \end{pmatrix}$, which has the dimension of an energy, corresponds to a direct Coulomb scattering between the "in" excitons (i, j) and the "out" excitons (m, n), the excitons m and i being made with the same electron-hole pair (see fig.(1a)). It precisely reads

$$\xi_{\rm dir}\begin{pmatrix} n & j \\ m & i \end{pmatrix} = \int d\mathbf{r}_e \, d\mathbf{r}_h \, d\mathbf{r}_{e'} \, d\mathbf{r}_{h'} \, \phi_n^*(\mathbf{r}_{e'}, \mathbf{r}_{h'}) \, \phi_m^*(\mathbf{r}_e, \mathbf{r}_h)$$
$$\left(V_{ee'} + V_{hh'} - V_{eh'} - V_{e'h}\right) \, \phi_i(\mathbf{r}_e, \mathbf{r}_h) \, \phi_j(\mathbf{r}_{e'}, \mathbf{r}_{h'}) \ , \tag{6}$$

where $\phi_i(\mathbf{r}_e, \mathbf{r}_h)$ is the i exciton wavefunction in $\mathbf{r}$ space and $V_{ij} = e^2/|\mathbf{r}_i - \mathbf{r}_j|$.

(ii) The second scattering, $\left[\lambda_h\begin{pmatrix} n & j \\ m & i \end{pmatrix} + \lambda_e\begin{pmatrix} n & j \\ m & i \end{pmatrix}\right] = 2\lambda_{mnij}$, which is dimensionless, is the sum of the possible carrier exchanges between the "in" excitons (i, j) and the "out" excitons (m, n), without any Coulomb process. In $\lambda_h\begin{pmatrix} n & j \\ m & i \end{pmatrix}$, the excitons exchange their hole, while in $\lambda_e\begin{pmatrix} n & j \\ m & i \end{pmatrix}$, they exchange their electrons (see fig.1b). $\lambda_h\begin{pmatrix} n & j \\ m & i \end{pmatrix}$ precisely reads

$$\lambda_h\begin{pmatrix} n & j \\ m & i \end{pmatrix} = \int d\mathbf{r}_e \, d\mathbf{r}_h \, d\mathbf{r}_{e'} \, d\mathbf{r}_{h'} \, \phi_n^*(\mathbf{r}_{e'}, \mathbf{r}_h) \, \phi_m^*(\mathbf{r}_e, \mathbf{r}_{h'}) \, \phi_i(\mathbf{r}_e, \mathbf{r}_h) \, \phi_j(\mathbf{r}_{e'}, \mathbf{r}_{h'}) \ , \tag{7}$$

while $\lambda_e\begin{pmatrix} n & j \\ m & i \end{pmatrix} = \lambda_h\begin{pmatrix} m & j \\ n & i \end{pmatrix} = \lambda_h\begin{pmatrix} n & i \\ m & j \end{pmatrix}$.

With these four commutators, we can calculate any physical quantities we wish. Indeed, physical quantities involving N excitons can always be written in terms of matrix elements like

$$\langle v|B_{m_N} \ldots B_{m_1} \, f(H) \, B_{i_1}^\dagger \ldots B_{i_N}^\dagger|v\rangle \ . \tag{8}$$

To calculate these matrix elements, we first push $f(H)$ to the right, to end with $f(H)|v\rangle = f(0)|v\rangle$, if the vacuum energy is taken as zero. This is done using eq. (1). In the simplest case, $f(H) = H$, we have

$$H\,B_i^\dagger = B_i^\dagger\,(H + E_i) + V_i^\dagger\ , \tag{9}$$

$V_i^\dagger$ being then pushed to the right according to eq. (2). Another $f(H)$ of interest can be $(a - H)^{-1}$. It appears in correlation effects or in optical nonlinearities, a then being $(\omega + i\eta)$. In this case, we use [4]

$$\frac{1}{a-H}\,B_i^\dagger = B_i^\dagger\,\frac{1}{a-(H+E_i)} + \frac{1}{a-H}\,V_i^\dagger\,\frac{1}{a-(H+E_i)}\ , \tag{10}$$

which follows from eq. (1), as easy to show by multiplying eq. (10) by $(a - H)$ on the left and $(a - H - E_i)$ on the right. A last $f(H)$ of interest is e^{-iHt}. It appears in problems dealing with time evolution. We then use [16]

$$\begin{aligned} e^{-iHt}\,B_i^\dagger &= B_i^\dagger\,e^{-i(H+E_i)t} + W_i^\dagger(t) \\ W_i^\dagger(t) &= -\int_{-\infty}^{+\infty}\frac{dx}{2i\pi}\,\frac{e^{-i(x+i\eta)t}}{x-H+i\eta}\,V_i^\dagger\,\frac{1}{x-H-E_i+i\eta}\ , \end{aligned} \tag{11}$$

which follows from eq. (10) when inserted in the integral representation of the exponential, namely

$$e^{-iHt} = -\int_{-\infty}^{+\infty}\frac{dx}{2i\pi}\,\frac{e^{-i(x+i\eta)t}}{x-H+i\eta}\ , \tag{12}$$

valid for t and η positive.

Once $f(H)$ is pushed to the right, we start pushing the B's to the right, according to eqs. (3,4), to end with $B_m|v\rangle = 0$ and $D_{mi}|v\rangle = 0$, which follows from eq. (4) applied to $|v\rangle$.

In problems dealing with many excitons, most of them are usually in the same state 0. It is then convenient to note that the iteration of eqs. (1,4) leads to [17]

$$\left[H, B_0^{\dagger N}\right] = NE_0\,B_0^{\dagger N} + NB_0^{\dagger N-1}V_0^\dagger + \frac{N(N-1)}{2}\,B_0^{\dagger N-2}\sum_{mn}\xi_{\text{dir}}\left(\begin{smallmatrix} n & 0 \\ m & 0\end{smallmatrix}\right)\,B_m^\dagger B_n^\dagger\ , \tag{13}$$

$$\left[B_m, B_0^{\dagger N}\right] = N\,B_0^{\dagger N-1}(\delta_{m,0} - D_{m0}) - N(N-1)\,B_0^{\dagger N-2}\sum_p\lambda_h\left(\begin{smallmatrix} p & 0 \\ m & 0\end{smallmatrix}\right)\,B_p^\dagger\ . \tag{14}$$

The real – challenging – difficulties in problems dealing with interacting excitons, is not so much the calculation of these matrix elements, which after all, can be tedious but is always conceptually trivial; the real difficulty is the determination of the appropriate

matrix elements we have to calculate to get the physical quantity we want. Indeed, all our background and practice in problems dealing with interactions, rely on perturbation theory, *i. e.*, on the possibility to isolate an interacting potential $V = H - H_0$, from the Hamiltonian. In the case of composite excitons, this is not possible: We cannot isolate from the Coulomb potential, the part corresponding to interactions *between* two excitons. Indeed, if we see them as made with (e,h) and (e',h'), we would say that the Coulomb potential between these excitons is $(V_{ee'} + V_{hh'} - V_{eh'} - V_{e'h})$, while if we see them as made with (eh') and $(e'h)$, this potential should be $(V_{ee'} + V_{hh'} - V_{eh} - V_{e'h'})$. Since the electrons (holes) are undistinguishable, there is no way to know ! Actually, the quantity $V_i^\dagger$, defined in eq. (1), plays the role of this interacting potential. It however has the dimension of a pair creation operator, this is why we have called it "Coulomb creation potential". To support this idea, we can note that the first terms of the right hand side of eqs. (9,10,11), which contain $(H + E_i)$, are in fact the "zero order terms" of the left hand side of these equations, *i. e.*, the terms in the absence of Coulomb interaction between the exciton i and the rest of the system.

The real difficulty to get the physics of interacting excitons, is thus to write the quantities we look for, not in terms of V, as we usually do, but in terms of the Hamiltonian H. The Hamiltonian *a priori* contains all the necessary informations on the interactions existing in the system. Consequently, even if this may not appear as obvious at first, there should be a way to write all quantities dealing with interacting excitons in terms of H only. (Note that, since there is no V, there is no H_0 – because its existence would mean that we could define V – so that there is no "unperturbed state" $|0\rangle$).

As an example of such a possible new writing, we have recently considered the lifetime and scattering rates of excitons [18]. In the usual case, when $H = H_0 + V$, they are given by the Fermi golden rule, which reads in terms of V. We have shown that the lifetime of an initial state $|\psi_0\rangle$ can also be written as

$$\frac{t}{\tau_0} = \langle\psi_0|\hat{H}|F_t(\hat{H})|^2\hat{H}|\psi_0\rangle \ , \tag{15}$$

where $\hat{H} = H - \langle\psi_0|H|\psi_0\rangle$, while $F_t(E) = (e^{-iEt} - 1)/E$, so that $|F_t(E)|^2 = 2\pi\, t\, \delta_t(E)$ with $\delta_t(E) = (\pi\, E)^{-1}\sin(Et/2)$ being the usual delta function of width $(2/t)$. It is easy to check that, for $H = H_0 + V$ and $|\psi_0\rangle = |0\rangle$, with $(H - E_n)|n\rangle = 0$, the Fermi golden rule follows from eq. (15): Indeed, as $\hat{H}|\psi_0\rangle = (1 - |0\rangle\langle 0|)\, V|0\rangle$ is just $\sum_{n\neq 0}|n\rangle\langle|V|0\rangle$, we

do recover

$$\frac{1}{\tau_0} = 2\pi \sum_{n\neq 0} |\langle n|V|0\rangle|^2 \ \delta_t(E_0 - E_n) \ . \tag{16}$$

Diagrams associated to this new many-body theory

The construction of a new many-body theory has for final goal, to write all processes in which enter interactions, at any order. Of course, to reach this goal, we always start in a pedestrian way, by generating the corresponding terms, through some kind of recursion procedure, in order to obtain their mathematical expressions. However, if we go to high orders, most of the time, these expressions become so complicated that we do not "see" their physical meaning anymore. This is why many-body theories are usually associated to diagrams which are just the visual representation of these mathematical quantities. These diagrams are really useful because they allow to "see" the physics involved in the corresponding terms, in an easy way.

In our new many-body theory for interacting excitons, enter two conceptually different scatterings. The direct Coulomb scattering between two excitons $\xi_{\text{dir}}\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right)$ is basically similar to the Coulomb scattering between free carriers. It gives rise to diagrams which look very much like the Feynman diagrams. On the opposite, the Pauli scatterings, $\left[\lambda_h\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right) + \lambda_e\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right)\right]$, are the conceptually new part of our theory. Although they appear as scatterings between two excitons (i,j), transforming them into (m,n), they originate from Pauli exclusion, which makes the excitons close-to-bosons only, as can be seen from eqs. (3,4). However, Pauli exclusion being N-body "at once" — an additional exciton having to be made with carriers in states different from *all* the ones involved in the excitons already present — it is clear that these Pauli scatterings between two excitons only, cannot be the quantities which appear at the end, in physical effects in which N excitons are involved. Nevertheless, they are quite convenient in the intermediate stages of the theory, because they allow to write recursion relations between scalar products of $N, (N-1), (N-2), \cdots$ exciton states.

Using eq. (14), we can in particular show that

$$\begin{aligned} A_{mi}^{(N)} &= \langle v|B_0^{N-1}B_m B_i^\dagger B_0^{\dagger N-1}|v\rangle \\ &= \delta_{m,i}\, A_{00}^{(N-1)} + (N-1)C_{mi}^{(N)} + (N-1)^2(N-2)^2 \sum_{pj} \lambda_{00ip}\,\lambda_{jm00}\,A_{pj}^{(N-2)}, \end{aligned} \tag{17}$$

where $A_{00}^{(N)} = N!F_N$ has already been calculated in references [5,7], while $C_{mi}^{(N)}$ is given by

$$C_{mi}^{(N)} = \delta_{0,i} A_{m0}^{(N-1)} - 2 \sum_p \lambda_{m0ip}\, A_{p0}^{(N-1)} - (N-1)(N-2)\, \delta_{m,0} \sum_p \lambda_{ooip}\, A_{p0}^{(N-2)} \ . \qquad (18)$$

If we iterate this equation and perform the sums over the dumb indices $(p, j, \ldots)$, through closure relations, we end with the "skeleton Pauli diagrams" [8] shown in fig.2.

This figure essentially says that the scalar product of one exciton m along with $(N-1)$ excitons 0 on the left, and one exciton i along with $(N-1)$ excitons 0 on the right, can be separated into a term in which the excitons 0 do not play any role, and terms in which one, two, ... excitons 0 out of N, exchange their carriers with the excitons m and i, in any possible way. The N-dependent prefactors are just the number of ways to choose these excitons 0 among $(N-1)$, on each side, while the sign is related to the parity of the number of exchanges involved in the diagram, as usual.

This diagrammatic representation of the matrix element of N-exciton states with one exciton different from 0 on each side, can be extended to any other scalar products [17] having more than one exciton different from 0. Although this extension may appear as reasonable, it did not appear to us obvious, at first ! It in fact relies on heavy calculations which use recursion relations similar to the one of eq. (17). This "reasonable" extension has actually been made possible due to the introduction of the "skeleton Pauli diagrams" [8], which allow to get rid of the relative positions of the Pauli scatterings λ_{mnij}, and which make disappearing all the awkward factors 2 due to $\lambda_e \binom{n\ 0}{m\ 0} = \lambda_h \binom{n\ 0}{m\ 0} = \lambda_h \binom{m\ 0}{n\ 0}$.

Using these "skeleton Pauli diagrams", in which can possibly enter some additional direct Coulomb scatterings between two exciton lines, due to possible H contributions to the matrix elements of interest, we can represent any quantity we wish, dealing with interacting excitons, in a transparent way.

Some applications of this new many-body theory

We have used this new many-body theory in a few problems in which interacting excitons are involved:

1) *Link with the effective bosonic Hamiltonian for excitons*

In order to make the link with the results obtained from the usual effective bosonic Hamiltonian for excitons, we have shown [1] that the scattering V_{mnij} which appears in

the interaction term of this Hamiltonian, $(1/2)\sum_{mnij} V_{mnij} \bar{B}^\dagger_m \bar{B}^\dagger_n \bar{B}_i \bar{B}_j$, where the $\bar{B}_i$'s are true boson operators, $[\bar{B}_i, \bar{B}^\dagger_j] = \delta_{i,j}$, is just

$$V_{mnij} = \xi_{\rm dir}\begin{pmatrix} n & j \\ m & i \end{pmatrix} - \sum_{pq} \xi_{\rm dir}\begin{pmatrix} n & q \\ m & p \end{pmatrix} \lambda_{pqij} \ . \tag{19}$$

Its replacement by

$$\xi_{\rm dir}\begin{pmatrix} n & j \\ m & i \end{pmatrix} - \frac{1}{2}\sum_{pq} \left[\xi_{\rm dir}\begin{pmatrix} n & q \\ m & p \end{pmatrix} \lambda_{pqij} + \lambda_{mnpq}\xi_{\rm dir}\begin{pmatrix} q & j \\ p & i \end{pmatrix}\right] \ , \tag{20}$$

would have allowed the effective Hamiltonian to be at least hermitian.

This hermiticity is however not enough to get correct results on interacting excitons, because $\xi_{\rm dir}$ and $\lambda_{e,h}$ are conceptually independent scatterings: They may appear in many other ways than the ones written in eqs. (19,20). In particular, in problems dealing with photons, $\lambda_{e,h}$ can appear alone [16,19], so that the corresponding terms are simply missed by *any* effective bosonic Hamiltonian, whatever these scatterings are. Indeed, V_{mnij} must have the dimension of an energy, while $\lambda_{e,h}$ is dimensionless. Moreover, Coulomb processes which may enter these physical effects, have to appear with energy denominators, in order to have the same dimension as the pure Pauli terms, which contain $\lambda_{e,h}$ only. In problems dealing with photons, these energies are in fact the exciton detunings, so that the Coulomb processes are negligible in front of the pure Pauli terms – missed by the effective bosonic Hamiltonians – in the large detuning limit.

2) *Scattering rates and lifetime of excitons*

We have shown that the lifetime of an exciton state in which all the excitons are in the same state 0, is related to the scattering rates towards other exciton states in which two excitons differ from 0, by the relation [18]

$$\frac{1}{\tau_0} = \alpha \sum_{(i,j)\text{couples}} \frac{1}{T_{ij}} \ , \tag{21}$$

where $\alpha = 1$ if the excitons are bosonized and $\alpha = 1/2$ if their composite nature is kept. This shows that it is impossible to find effective scatterings for the effective bosonic Hamiltonian, giving both the lifetime and scattering rates correctly.

This type of sum rule comes from closure relation. We have shown that, while the one for bosonized excitons is of course the standard closure relation for elementary particles,

$$I = \frac{1}{N!}\sum_{i_1,\ldots,i_N} \bar{B}^\dagger_{i_1}\cdots\bar{B}^\dagger_{i_N}|v\rangle\langle v|\bar{B}_{i_N}\cdots\bar{B}_{i_1} \ , \tag{22}$$

a closure relation also exists for composite excitons, in spite of the fact that these states form a non-orthogonal overcomplete set [20]. It actually reads [21]

$$I = \frac{1}{(N!)^2} \sum_{i_1,\ldots,i_N} B^\dagger_{i_1} \cdots B^\dagger_{i_N} |v\rangle\langle v| B_{i_N} \cdots B_{i_1} \ . \tag{23}$$

Consequently, all sum rules will appear differently for exact and boson excitons, making the corresponding quantities irretrievably different.

3) *The trion as an exciton interacting with a carrier*

This new many-body theory for excitons interacting with excitons can be adapted to excitons interacting with free carriers [22-24].

The simplest of these problems is surely the trion: The exciton interacts with one electron either by Coulomb interaction or by electron exchange. Using this approach, we can calculate the trion oscillator strength in a quite easy way [25]. We have shown that it is one exciton volume divided by one sample volume smaller than the exciton one, making the trion impossible to see in the linear response of a macroscopic semiconductor sample.

This approach should allow us to face the much harder problem of the linear response of a doped semiconductor quantum well. Our old work on this problem [26] is quite unsatisfactory, not at all because Coulomb screening is missing, but because the electron-electron repulsion has been neglected. In order to include it, *i. e.*, in order to study the rearrangement of a Fermi sea to the sudden appearance of an exciton – not simply to a hole, as we did – we have to handle not only Coulomb interaction between the exciton and the Fermi sea electrons, but also all possible electron exchanges. This again makes our new many-body theory for interacting excitons rather unavoidable.

4) *Optical nonlinearities*

The optical nonlinearities come from interactions with the virtual excitations induced in the matter by the photons. In the case of semiconductors, these excitations are the excitons. Consequently, in order to derive the optical nonlinearities in semiconductors properly, it is necessary to know how to handle interactions between excitons in a secure way. This is why our new many-body theory for composite excitons is going to be all the most useful for semiconductor optical nonlinearities.

We have first reconsidered the calculation of the third order susceptibility $\chi^{(3)}$ [16]. This calculation contains a major difficulty, unsolved before our work, which comes from the fact that $\chi^{(3)}$ appears as the sum of 16 terms which all increase linearly with the sample

volume. Of course, as the susceptibility is an intensive quantity, these volume-linear terms have to cancel. However, this cancellation has to be shown very carefully in order to get the correct volume-free terms which remain once the volume-linear terms are removed. Our many-body theory allows to calculate these 16 terms explicitly and to combine them in order to show that the volume-linear terms do disappear as expected. However, this clean cancellation in fact generates volume-free terms missed by the previous approaches, which come from pure Pauli interactions between the virtual excitons coupled to the photons. Again, these missed pure Pauli terms are the dominant ones at large detuning.

Another nonlinear effect in semiconductor physics is the exciton optical Stark effect [27]. The seeds of our new many-body theory were actually in our old works on this effect, in particular the Coulomb creation potential $V_i^\dagger$. The introduction of the two scatterings ξ_{dir} and $\lambda_{e,h}$ however make the study of this exciton optical Stark effect quite easy, in particular if we are interested in subtle polarization effects.

Our many-body theory is also very convenient to study spin manipulations by laser pulse [19]. We have recently predicted the spin precession of an electron bound to an impurity, when the sample is irradiated by a pump beam tuned in the transparency region. This precession, which is due to exchanges between the electron bound to the impurity and the electron of the virtual exciton coupled to the pump photon, lasts as long as the laser pulse, so that it can be used for ultrafast spin manipulation.

Conclusion

- We have developed a new many-body theory adapted to interactions between composite excitons.

- It relies on two "scatterings": One is associated to Coulomb processes, the other is associated to carrier exchanges induced by Pauli exclusion. This Pauli scattering can appear alone or mixed with Coulomb processes, in many more subtle ways than the one appearing in the effective bosonic Hamiltonian for excitons. When it appears alone, the corresponding terms are *a priori* missed by the effective bosonic Hamiltonians, whatever are the scatterings they use. These missed terms are often the dominant ones, due to dimensional arguments.

- We have generated a diagrammatic representation of this theory. The direct Coulomb

scattering gives rise to diagrams similar to the Feynman diagrams. On the opposite, the Pauli scattering gives rise to diagrams which are conceptually new: Pauli exclusion being N-body "at once", the corresponding "skeleton Pauli diagrams" show all possible carrier exchanges between 2,3,4. . . excitons, in a quite transparent way.

- This new many-body theory is going to be of great interest for all problems dealing with interactions between excitons and between excitons and free carriers, as well as for all optical nonlinearities in semiconductors, because, for the first time, it allows a clean treatment of the possible carrier exchanges.

- This new theory is also going to be of importance in many other fields than semiconductor physics, because essentially all particles considered as bosons, are in fact composite bosons.

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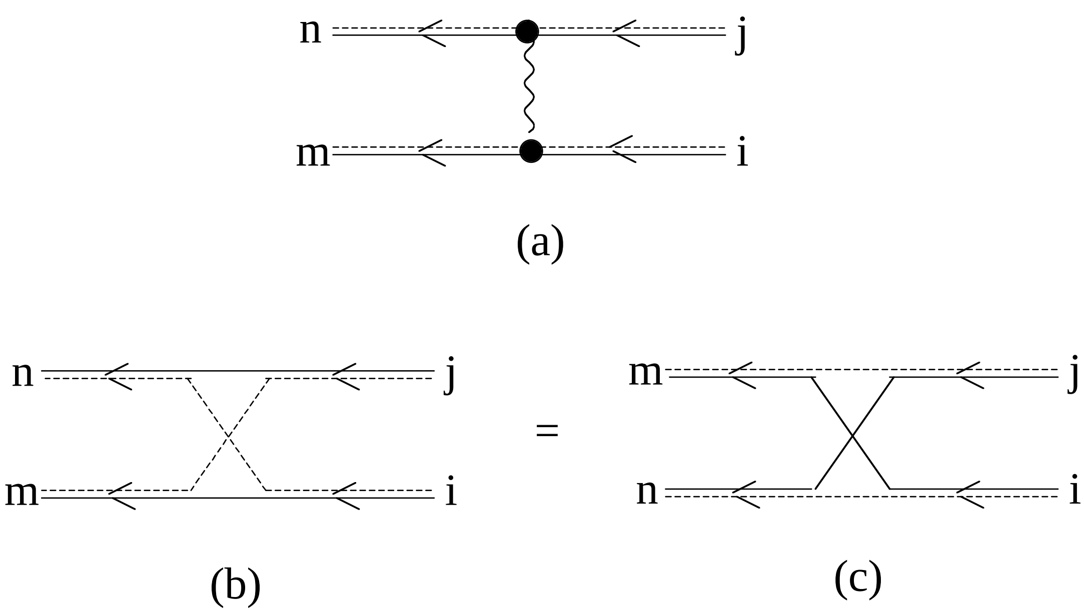

Figure 1: (a) Direct Coulomb scattering $\xi_{\mathrm{dir}}\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right)$ between the "in" excitons (i, j) and the "out" excitons (m, n). (b) Exchange scattering $\lambda_h\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right)$ in which the excitons exchange their holes. In (c), which shows $\lambda_e\left(\begin{smallmatrix} m & j \\ n & i \end{smallmatrix}\right)$, they exchange their electrons. Note that $\lambda_h\left(\begin{smallmatrix} n & j \\ m & i \end{smallmatrix}\right) = \lambda_e\left(\begin{smallmatrix} m & j \\ n & i \end{smallmatrix}\right)$.

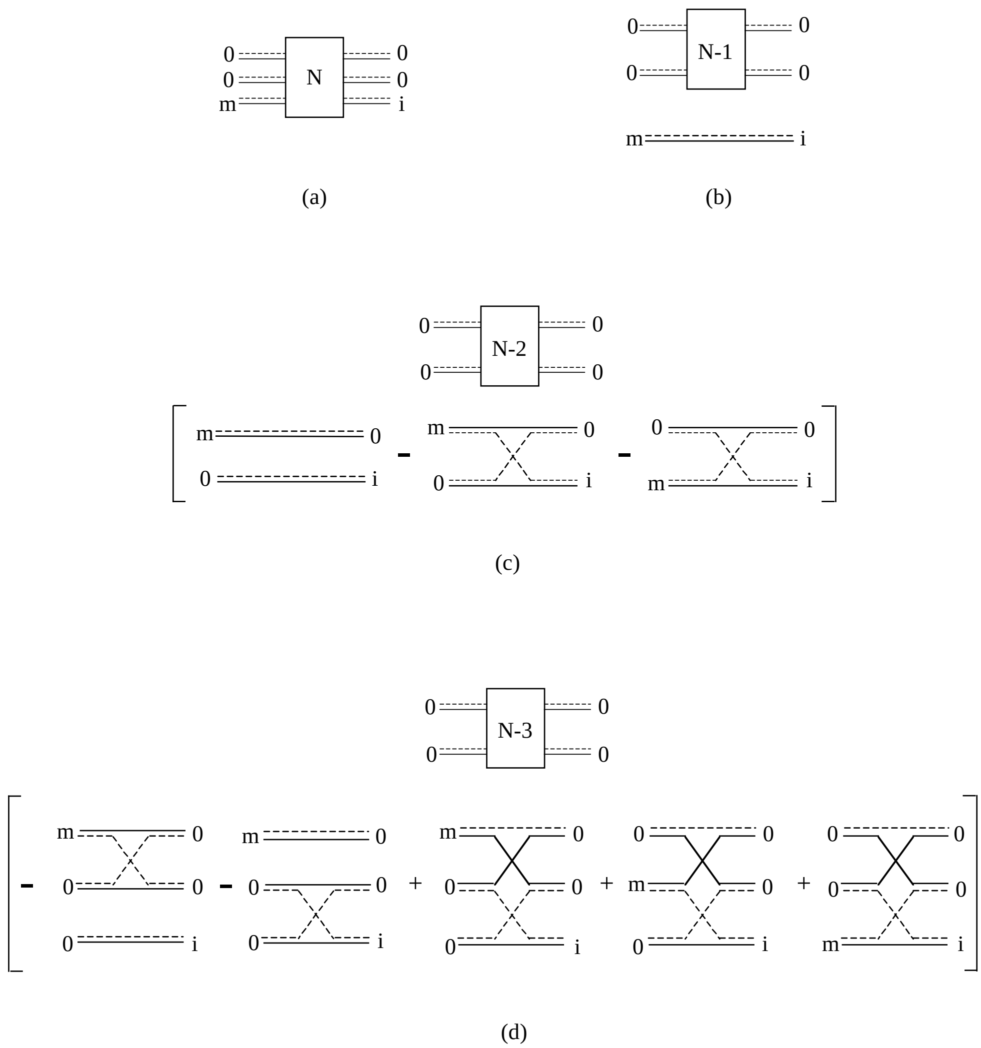

Figure 2: Diagrammatic expansion in "skeleton Pauli diagrams" of the scalar product $A_{mi}^{(N)}$, defined in eq. (17) and shown in (a), in which all excitons but one are in the same state 0. In (b), the (m, i) excitons do not exchange any carrier with the sea of $(N-1)$ excitons 0. In (c), one exciton 0 is involved in carrier exchanges with (m, i). In (d), two excitons 0 are involved; and so on... The term in which p excitons 0 are involved is equal to the scalar product, $A_{00}^{(N-1-p)}$, of the remaining $(N-1-p)$ excitons 0, shown as a box with $(N-1-p)$ inside, multiplied by all possible carrier exchanges between the (m, i) excitons and the p excitons 0, topologically connected or not, but without diagrams with excitons 0 alone. The prefactor, not shown in the figure, is the number of ways to choose the p excitons 0 among $(N-1)$, on each side, *i. e.*, $[(N-1)!/(N-1-p)!]^2$, the sign being linked to the parity of the number of carrier exchanges, as usual.